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Water distribution network analysis accounting for different background leakage models

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Abstract

This contribution analyzes the implementation of two widely used literature leakage models within an advanced pressure-driven hydraulic simulation model (WDNetXL system [1]). The used leakage modelling approaches are that introduced by Germanopoulos [2] and that proposed by Van Zyl and Cassa [3] based on experimental evidences under the assumption of linear elastic behavior of pipes. The modelling approaches are discussed from different perspective and tested on the hydraulic analysis of a literature network.

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Keywords: Background leakage modelling; Hydraulic analysis; Water distribution network.

1. Introduction

In recent decades, pressure management in water distribution network (WDN) has been recognized as essential for effective leakage management. Benefits of managing pressures range from water loss reduction, limiting the deterioration of network components (joints, valves, pipes, etc.), and to reducing frequency of new leaks in the network [4][5]. At the same time, the general development of information technology and the increase of computational capabilities coupled with the increasing interest for WDN analysis and management of water utilities, have led to significant advancement in WDN hydraulic simulation (i.e., WDN pressure-driven analysis).

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In this scenario, effective modelling of water leakages is essential for an accurate analysis of pressure management strategies through WDN simulation models and for efficient model calibration for existing WDNs [4][6], as well as for leakage detection techniques based on inverse analysis [7].

The starting point is that water leakages in WDN are directly related to pressure as well as to age and material of WDN elements (joints, valves, pipes, etc.). Therefore, a number of numerical models were developed starting from experimental observations and assuming the validity of the Torricelli law in the following form, which is valid for a single leak orifice,

$$Q_{leak} = C_d A\sqrt{2gP} \tag{1}$$

where Q_{leak} is the leakage flow rate through a hole of area A under a pressure head P; g is the acceleration due to gravity. The coefficient C_d is introduced in order to take into account that for real fluid the cross sectional area of the water jet is less than the orifice area, due to frictional energy losses.

However, evidences from field studies and implementation of pressure management strategies all over the world showed that the leakage rate into a WDN can be more sensitive to pressure than what said by the Torricelli law, with exponents for pressure head P in the range 0.5 to 2.95 [8]. This has led to assume that the effective area A_E can be pressure-dependent, in relation to the presence of various types of leaks, affecting different elements of the network, with materials of different deformability. Therefore, from the hydraulic modelling perspective, given the complexity of representing physically such a large number of individual leaks within the WDN (i.e., understanding the behavior of leaks individually), it is very important to rely on a model for leakage representation within the hydraulic simulation model, which is efficient for pressure management strategies and network model calibration.

The aim of this contribution is to compare and discuss two widely used leakage models from literature, also based on their implementation within a pressure-driven hydraulic simulation model, i.e., the WDNetXL system [1]. The used leakage modelling approaches are that introduced by [2] and that proposed by [3] based on experimental evidences under the assumption of linear elastic behavior of pipes.

2. Background on leakage modelling

Water distribution system losses may be classified as due to background losses (e.g., from joints, fittings, and small cracks), reported bursts, and unreported bursts [9]. Bursts are intended as major water outflow events that are usually reported to water utilities and repaired since they are likely to produce major service disruptions. For this reason burst are commonly considered as accidents whose impact on WDN can be limited by improving active leakage control and the efficiency of detection and repair actions. Vice versa, background leakages are intended as outflows running from small cracks, holes, deteriorated joints or fittings, occurring along pipes. As diffused water outflows, background leakages do not result into evident and quick pressure drops through the network, thus they are not reported and run for longer time, producing relevant impact in terms of WDN water lost volumes. For this reason background leakages can be reduced by planning medium-long term interventions for asset rehabilitation and pressure management. Therefore, the following formulations are referred to background leakages modelling.

In both cases, breaks along pipes can be assimilated firstly to orifices, and for them, originally, the Eq. (1) was considered valid. However, from several studies on real WDNs, it has been shown that the Torricelli law does not provide a satisfactory model for the relationship between leakages with pressure within a WDN.

As a result, leakage practitioners and water utilities adopted a more general leakage equation, which formally was proposed by [1][10] in the form of a power equation,

$$Q_{leak} = \beta P^{\alpha} \tag{2}$$

where P is the average pressure head on the element/segment of the WDN considered (e.g., the generic pipe, the pressure zone, the whole network). Variables α and β are two leakage model parameters, that represent the influence of some factors on the relationship leakage/pressure. Parameter β can represent the pipe deterioration over time, thus it depends on both pipe characteristics (pipe age, diameter, and material) and various external factors (mainly the average pressure, but also other environmental conditions, traffic loading, external stress and corrosion, etc.). In contrast, α is a function of pipe characteristics only (material and elasticity) [11]. In general, β is more closely related to the number of leaks (or leakage area) per unit of pipe length while α is more strongly related to the type of leakage (therefore to the hydraulics of leakage) as governed by the pipe material [12]. For this reason, changes in β need to be determined for the specific system, i.e. by model calibration, while the most part of experimental studies have focused mainly on leakage parameter α.

Field tests have found system values of α substantially higher than 0.5, as for example Ogura (1979), who isolated WDN segments using isolation valves, using a pump and measuring equipment to estimate the pressure-leakage relationship for each pipe WDN segment. Leakage parameters α returned varied between 0.65 and 2.1, with an average of 1.15. Later on, this study was confirmed by works of [13] and [14], whose obtained the value of α =1.18 from field data. From his study on field data [12] returned a range of α values ranging from 0.50 to 2.50, depending on the mixture of leaks and the dominant type of leaks: (i) simple holes (α = 0.5); (ii) longitudinal split that opens in one dimension, [15] (α = 1.5); (iii) linear-radial opening (α =2.0−2.5). Plastic pipes exhibit higher values of α because of their propensity to have longitudinal splits. Details of several field study of this parameter can be found in [16], with exponent varying between 0.5 and 2.95.

Laboratory test investigated also the main parameters influencing the leakage parameter α for the single hole, which mainly results dependent on pipe material behavior and circumferential rigidity [17]. Greyvenstein and Van Zyl [18] investigated the pressure-leakage relationship on individual leaks in pipes showing that for round holes the leakage parameters α is close to 0.5, regardless to the pipe material or hole size. The relationship is complicated for pipe with plastic behavior through hysteresis and plastic deformation as documented by [19][20].

The value of the leakage parameter α in Eq. (2) can be also described using the fixed and variable area discharge (FAVAD) approach proposed by [15]. This approach assumed that some leaks are rigid, i.e., burst losses through a constant area hole (α = 0.5), while others will expand with increasing pressure, i.e., background losses through an area that changes linearly with pressure (α = 1+0.5 = 1.5). May [15] proposed a combined leakage equation in the following form, under a pressure head P on the single leak,

$$Q_{leak} = \beta_1 P^{0.5} + \beta_2 P^{1.5} \tag{3}$$

where β_1 and β_2 are the coefficients characterizing the rigid leaks and variable-area leaks, respectively, into the whole WDN. The formulation prosed by [15] became prominent in the sector, as it explained the wide range of pressure-leak flow relationships measured internationally on field. Therefore, the FAVAD concept was later adopted and recommended for international use by the IWA Water Losses Task Force.

Van Zyl and Cassa [3] proposed to complete the expression in Eq. (3), based on previous experimental studies of the same authors under the assumption of linear elastic behavior of the pipe. From the experimental evidence that the areas of various types of leak openings (round holes and longitudinal, circumferential, and spiral cracks) varied linearly with pressure regardless to pipe dimensions, material, and loading conditions, they proposed the following expression for the area A of any leak undergoing elastic deformation,

$$A = A_0 + mP \tag{4}$$

where A_0 = initial leak area (under zero pressure conditions); and m = head-area slope.

Replacing this relationship into Eq. (1), is possible to write,

$$Q_{leak} = C_d\sqrt{2g}\left(A_0 P^{0.5} + mP^{1.5}\right) = \left(C_d A_0\sqrt{2g}\right)P^{0.5} + \left(C_d m\sqrt{2g}\right)P^{1.5} = \beta_1 P^{0.5} + \beta_2 P^{1.5} \tag{5}$$

in which parameters β_1 and β_2 in Eq. (3) have a clear expression due to the findings of [3]. In particular, Eq. (4) states that all leaks will increase in area with increasing pressure. For leaks with small head-area slopes, the first term of Eq. (5) is likely to be dominant, resulting in an effective leakage exponent of 0.5. Conversely, for flexible leaks with high head-area slopes, the second term of the equation will be dominant, resulting in leakage exponents of 1.5. From Eq. (5), it is evident that, under elastic conditions, the pressure response of a leak can be fully characterized by knowing its initial area A_0 and head-area slope m.

This is not an easy task in real systems, where representative values for all (or groups of) pipes need to be assumed (or estimated), as explained in the following section.

3. Analysis of background leakage models for hydraulic simulation

The analysis of leakage models here proposed is aimed at their implementation within a pressure-driven WDN simulation model.

Many of the formulas set out in the above-mentioned studies have been developed for the individual leak and then eventually applied extensively on the whole network based on real data measured or estimated in some way. This have laid a good foundation for leakage analysis, but it is difficult to apply formulas given as Eq. (1) or Eq. (5) to system-wide leakage analysis using a hydraulic model. This is simply due to the unknown types of leakages, thus the values of the hole area, the head-area slope, etc. Eq. (2) was efficiently assuming uniformly distributed leakages along pipes [2][10][11], still remaining open the issue of defining the coefficients α and β.

For these reason, this work aims to compare the two models in Eq. (2) and Eq. (5) in order to investigate the role of all the variables involved, and the influence of their variability in the context of the WDN hydraulic simulation model. However, in the following, for the sake of simplicity, all analysis will be showed considering the single leak; then, discussion will regards also the generalization to the entire WDN.

Starting from Eq. (1), assuming the leak hole area A as dependent on P through a power function (exponent γ), is possible to write,

$$Q_{leak} = C_d A\sqrt{2g}P^{0.5} \xrightarrow{A=A_0P^{\gamma}} Q_{leak} = \left(C_d A_0\sqrt{2g}\right)P^{\gamma}P^{0.5} \tag{6}$$

This formulation can be equal to Eq. (2) assuming the exponent $\gamma = \alpha - 0.5$. In particular, observing the left term in brackets on the right of Eq. (5), is possible to write that,

$$Q_{leak} = \beta_1 P^{\alpha-0.5}P^{0.5} = \beta_1 P^{\alpha} \tag{7}$$

Therefore, both Eq. (5) and Eq. (7) assume that leakage flow rate can change with pressure, according two similar approaches; in fact, assuming that a certain leakage flow rate can be expressed according to Eq. (7) or Eq. (5), it is possible to write

$$\beta_1 P^{0.5} + \beta_2 P^{1.5} = \beta_1 P^{\alpha} \tag{8}$$

obtaining the expression of the exponent α as

$$\alpha = \frac{1}{2} + \frac{\ln\left(1+\dfrac{\beta_2}{\beta_1}P\right)}{\ln P} \tag{9}$$

From a numerical standpoint, the main difference between Eq. (5) and Eq. (7) is that the former relies on two coefficients (β_1 and β_2), one for the leakages in rigid pipes and one for the leakages in flexible pipes in the WDN, that can be determined by means of model calibration and/or component analysis. The latter needs to define an exponent α that is somewhat representative of the hydraulics of the phenomenon. Actually, the exponent α of the leakage model in Eq. (7) encloses the balance between the burst (α = 0.5) and background (α > 0.5) (or between rigid and flexible pipes) and can be determined by means of model calibration and/or component analysis. This dependency is evident in Eq. (9), where, additionally, also the pressure head P has a relevant role.

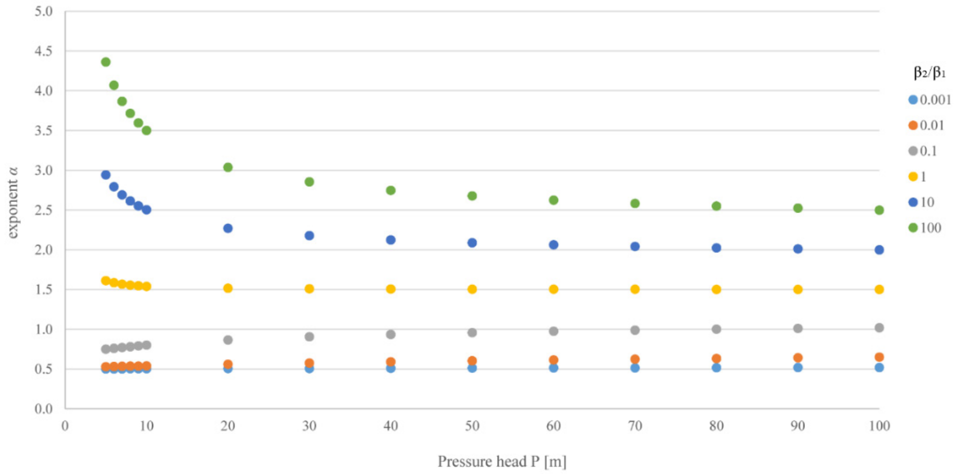

Fig. 1. Variability of exponent α according to Eq. (9).

In order to understand, how different combinations of such values can influence values of exponent α, the diagram in Fig.1 has been developed by varying the ratio β_2/β_1 between [10^{-3}; 10^{2}] and P between [5m; 100m], thus assuming that the behavior of the pipes (rigid or flexible) are known/defined by means of the coefficients of Eq. (5).

From Fig.1 is clear that for low values of the ratio β_2/β_1, which means that leaks in rigid pipes are dominant on the leaks in flexible pipes (with variable leak area), the exponent α is equal to or very close to 0.5, independently to pressure head. When the leaks in flexible pipes are the 10% of leaks in rigid pipes ($\beta_2/\beta_1 = 0.1$) the exponent $\alpha = 1$ thus the relation between leakage flow rate and pressure became linear according to Eq. (7). According to the evidences of the field studies above reported, when leaks with pressure-dependent leak area (i.e., flexible pipes) are dominant the exponent α is approaching 2.5. Moreover, diagram in Fig. 1, can justify the upper bound (2.95) of exponents reported in [16], that could be admissible for $\beta_2/\beta_1 = 100$ and average pressure heads between 30 and 40 m. Finally, it is clear that, except for deficit conditions ($P < 10$m), pressure head has not relevant influence on the value of exponent α, with a small exception in case of predominant flexible pipes.

From Eq. (8) is possible to write also the following expression,

$$\frac{\beta_2}{\beta_1} = P^{-1}\left(P^{\alpha-0.5} - 1\right) \tag{10}$$

Thus, assuming that the behavior of the pipes (rigid or flexible) are known/defined by means of the coefficients of Eq. (7), it is possible to analyze the influence of exponent α on the values of β_2/β_1 by varying the average pressure head. The results of such analysis are reported in Fig. 2, which has been built by varying exponent α between [0.6; 2.5] and P between [5m; 100m].

As clear from Fig. 2 there is a great variability of the ratio β_2/β_1, thus of the balance between rigid and flexible pipes in the WDN, with pressure head given a certain value of exponent α. This means that, for leakage modelling, the definition (or assumption of a prior, in case of model calibration) of the exponent α does not entirely represent the balance between rigid and flexible pipes in the WDN, which is also influenced by the average pressure head on the WDN (or considered segment). Therefore, it could be more reliable to use a model like that in Eq. (5), for which the calibration of parameters is more reliable and mainly independent from the average pressure head (see Fig. 1).

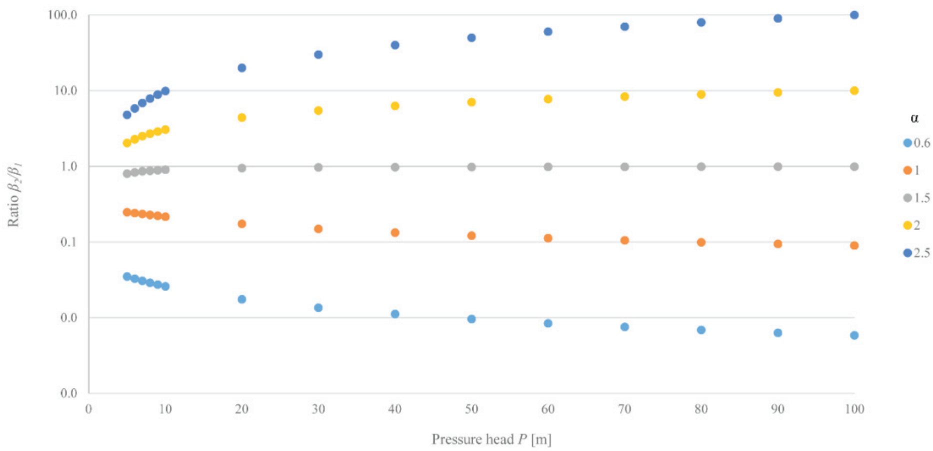

Fig. 2. Variability of ratio β_2/β_1 according to Eq. (10).

4. Models implementation in a pressure-driven simulation model

Here a simple case study is proposed aiming to show results of an extended period pressure-driven analysis of a WDN implementing the leakage models in Eq. (5) and Eq. (7). The analyzed network is Apulian Network (see Fig. 3), which is widely used in the literature as benchmark network in several applications [21].

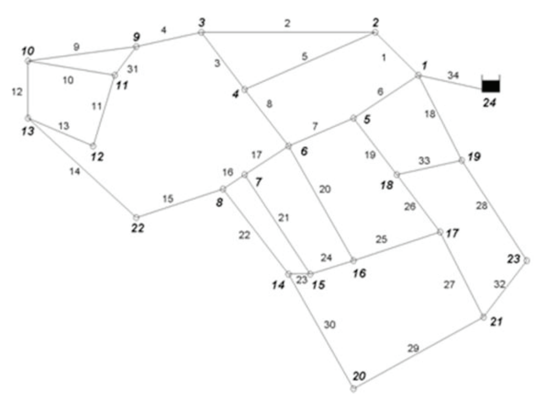

Fig. 3. Apulian network layout.

The hydraulic simulation is performed using the WDNetXL system [1], which is an integrated software framework composed by several MS-Excel® add-ins for WDN analysis, planning and management, combining advanced and robust WDN hydraulic simulation with topological analysis and optimization strategies to support technicians for complex WDN analysis, design and management problems. One of the key feature of the hydraulic simulation module in WDNetXL is the possibility to perform the pressure-driven analysis of the WDN, including the simulation of background leakages, and the definition of several components of the demand in each node [6].

In particular, background leakages can be modelled using different formulations, as those in Eq. (5) and Eq. (7). Note that, in WDNetXL, background leakages are computed at pipe level as a summation of small outflows due to cracks, small water losses from fitting, etc.. For the simulation problem solution, the model evaluates the pressure head P as the mean pressure along the each pipe ($P_{k,mean}$), considering also the pipe length L_k, and the model formulations becomes:

$$d_k^{leaks}\left(P_{k,mean}\right)=\begin{cases}\beta_{1,k}L_kP_{k,mean}^{0.5}+\beta_{2,k}L_kP_{k,mean}^{1.5} & P_{k,mean}>0\\ 0 & P_{k,mean}\le 0\end{cases}$$

$$d_k^{leaks}\left(P_{k,mean}\right)=\begin{cases}\beta_{1,k}L_kP_{k,mean}^{\alpha_k} & P_{k,mean}>0\\ 0 & P_{k,mean}\le 0\end{cases} \quad (11)$$

respectively for Eq. (5) and Eq. (7), where d_k^{leaks} = background leakages outflow along the k^{th} pipe.

The analysis here presented consists in comparing the background leakages volumes computed during the day (24 steady state simulation, one for each hour in a day) by the WDNetXL simulation model, assuming different balances between rigid and flexible pipes in the network, and different average pressure heads. The balance between rigid and flexible pipes in the network is represented by the exponent α in Eq. (7) and by the ratio β_2/β_1, as calculated by Eq. (10) after evaluating the average pressure head in the network.

Table 1. Leakage coefficients used for the application.

α	0.6	0.6	1.8	1.8
β_1	$5.00\cdot10^{-7}$	$5.00\cdot10^{-7}$	$7.00\cdot10^{-8}$	$7.00\cdot10^{-8}$
Average Pressure head [m]	20.35	67.69	11.66	14.23
β_2	$8.638\cdot10^{-7}$	$3.872\cdot10^{-9}$	$1.402\cdot10^{-7}$	$1.503\cdot10^{-7}$
β_2/β_1	0.017	0.008	2.004	2.148

The first computation assumes that rigid pipes are predominant and thus the exponent α = 0.6. All the used values are reported in Table 1. Once, the 24-hours simulation has been performed, the average pressure head returned was P = 20.35 m and consequently the value of β_2 has been calculated by Eq. (10), see Table 1, leading to a ratio β_2/β_1 = 0.017. Fig. 4 (left side), reports the background leakage volumes for the 24 hours calculated by both the analyzed leakage models in this case.

The same asset conditions has been analyzed assuming a higher average pressure (P = 67.69 m), returning the background leakage volumes for the 24 hours in Fig. 4 (right side), and a smaller ratio β_2/β_1 = 0.008, i.e. halved compared to the previous. This confirm that the only definition of α does not characterize the average behavior of the network leakages (rigid/flexible), and pressure head needs to be accounted for.

In terms of leakage volumes, Fig. 4 shows that there are small differences between the two models, since Eq. (5) returns higher volumes during the night hours (highest pressure) and lower during peak hours (low pressure), even if the total daily leakage volumes computed for the two pressure conditions are not actually different (see Table 2).

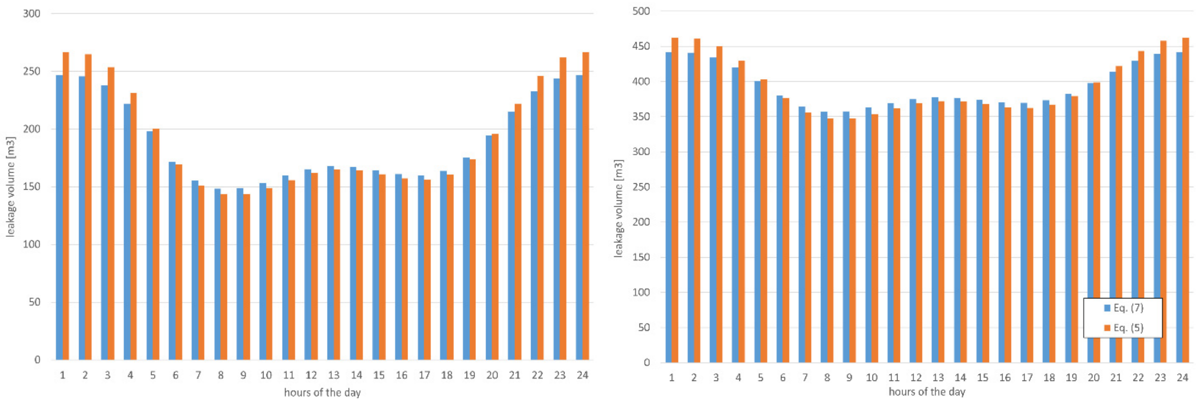

Fig. 4. Background leakage volumes for the 24 hours calculated using low pressure (left) and high pressure (right) - α = 0.6.

The second computation considers that in the WDN there are more flexible than rigid pipes, thus starting from an exponent α = 1.8. Note that for this second application customers nodal demands has not been changed, and thus the coefficient β_2 is lower than in the first application in order to contain the leakage to realistic values.

Once, the 24-hours simulation has been performed, the average pressure head returned was P = 11.66 m and consequently the value of β_2 has been calculated by Eq. (10), see Table 1, leading to a ratio β_2/β_1 = 2.004. Therefore, given the exponent α and the average pressure in the WDN the flexible pipes are the two thirds of the total. Fig. 5 (left side), reports the background leakage volumes for the 24 hours calculated by both the analyzed leakage models in this second case. The same asset conditions has been analyzed assuming a higher average pressure (P = 14.23 m), returning the background leakage volumes for the 24 hours in Fig. 5 (right side), and a higher ratio β_2/β_1 = 2.148.

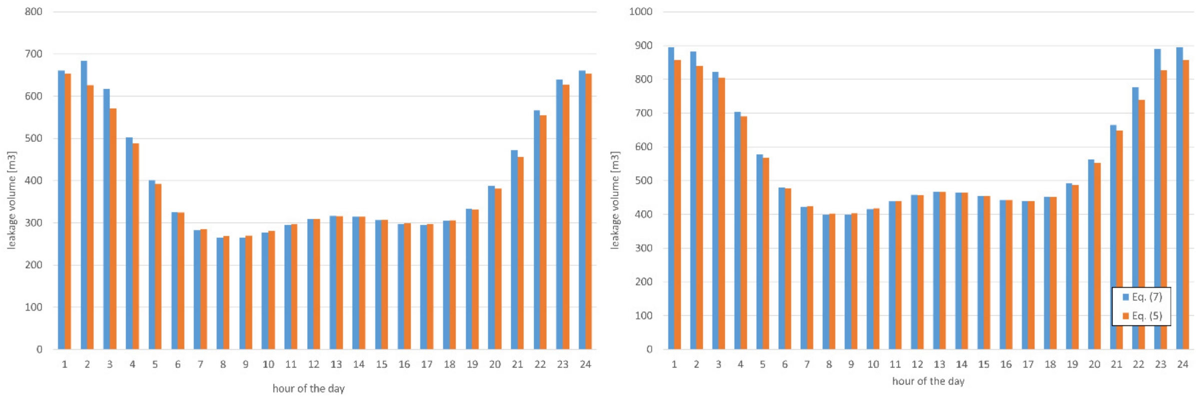

Fig. 5. Background leakage volumes for the 24 hours calculated using low pressure (left) and high pressure (right) - α = 1.8.

In this case also, the representation of the average network leakages behavior of the WDN cannot be defined reliably only by the exponent α. In terms of leakage volumes, Fig. 5 shows that there are small differences between the two models, particularly in the night hours when, unlike to the previous case, Eq. (7) returns higher volumes than Eq. (5), as confirmed by Table 2 that report the total daily leakage volumes computed for the two pressure conditions.

Table 2. Daily leakage volumes computed in different conditions.

	$\alpha = 0.6$		$\alpha = 1.8$	
	Average Pressure head [m]		Average Pressure head [m]	
	20.35	67.69	11.66	14.23
Eq. (7)	4544 m^3	9450 m^3	9776 m^3	13894 m^3
Eq. (5)	4619 m^3	9484 m^3	9609 m^3	13605 m^3

5. Conclusions

This contribution compared and discuss two widely used leakage models from literature, that introduced by [2] and that proposed by [3] based on experimental evidences under the assumption of linear elastic behavior of pipes. These models has been described with respect to the international research on leakage modelling and tested within an advanced pressure-driven hydraulic simulation model, i.e., the WDNetXL system [1].

A noteworthy evidence is that, in case of application of the model from [2], see Eq. (7), the definition of the balance between rigid and flexible behavior of leakages in the WDN depends on the definition of the model parameters (i.e. the exponent α and the coefficient β_1) and on the average pressure head of the considered element (entire network, WDN segment, pipe). This means that, for leakage modelling, the definition (or assumption of a prior, in case of model calibration) of the only exponent α is not enough to represent the probable leakage behavior in the WDN, since it depends also on the average pressure head, and thus on other factors as customers demand patterns, pumping, etc.. This fact is likely to result into difficulties in converging towards stable calibration results and, commonly, is overcome by bounding the numerical values of α in a narrow range.

Parameters β_1 and β_2 are likely to encompass all pipe features that represent the likelihood of failure for rigid and flexible components, and their ranges of variation are wider than α, depending on pipe conditions. For this reason, parameters β need to be calibrated minimizing the mismatch between model prediction and field observations. Similarly to unit pipe hydraulic resistance [22] the background leakage model coefficients β_1 and β_2 also encompasses all uncertainties about, for instance, the actual pipe deterioration conditions, the characteristics of surrounding soil, the fatigue effects due to possible pressure oscillation (e.g. due to transients that are neglected in steady-state hydraulic simulation). Since the estimate of such parameter by direct pipe inspection is technically not feasible, all these considerations suggest that a credible calibration of parameters β should reflect similar propensity to leak of groups (i.e. cohorts) of pipes sharing similar pipe features. From this point of view, it could be more consistent to use a model like that proposed by [3], see Eq. (5), for which the calibration of parameters is more reliable and mainly independent from the average pressure head (see Fig. 1). However, results from the implementation within the pressure-driven simulation model, see Table 2, show that there is no big difference between the considered models in terms of total daily leakage volumes.

Further studies will investigate the behavior of the analyzed models for real networks modelling in different working conditions, and develop new functional relationships for WDN leakage models based on real data.

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